



Review

Review of utilization of genetic algorithms in heat transfer problems

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ABSTRACT

This review presents when and how Genetic Algorithms (GAs) have been used over the last 15 years in the field of heat transfer. GAs are an optimization tool based on Darwinian evolution. They have been developed in the 1970s, but their utilization in heat transfer problems is more recent. In particular, the last couple of years have seen a sharp increase of interest in GAs for heat transfer related optimization problems. Three main families of heat transfer problems using GAs have been identified: (i) thermal systems design problems, (ii) inverse heat transfer problems, and (iii) development of heat transfer correlations. We present here the main features of the problems addressed with GAs including the modeling, number of variables, and GA settings. This information is useful for future use of GAs in heat transfer. Future possibilities and accomplishments of GAs in heat transfer are also drawn.

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1. Why review GAs in heat transfer?

Genetic Algorithms (GAs) were mostly developed in the 1970s as an optimization toolbox, though some work had already been done in the field of evolutionary computation. In 1967, Bagley [1] introduced the words “genetic algorithm” and published the first application of GAs. However, the first main works related to GAs are attributed to Holland [2] and De Jong [3], in 1975. In the 1980s, Grefenstette [4], Baker [5] and Goldberg [6] contributed to significant advancements in GAs. Ref. [6] presents a good picture of the state of art in 1989. A more complete history of GAs and other evolutionary computation methods is given in [7].

However, the interest in and the utilization of GAs in the field of heat transfer is more recent. This is probably due to the fact that for most numerical problems in which the heat transfer community is interested in, computational times are typically long. In the GA optimization procedure, several simulations typically need to be performed. When, for example, the simulation of a design involves CFD analysis, the overall computational time required for the GA to run could be prohibitive. Nevertheless, GAs began to be used in heat transfer approximately in the mid-1990s, timidly at first, but more and more regularly nowadays. As noted Queipo et al. back in 1994 [8], “the heat transfer community can expect to see a significant increase in pioneering applications of such methodologies [GAs] to many complicated thermoscience problems admitting optimization in some sense. These exciting applications are being facilitated by the increased availability of high performance computers, distributed computing environments and improved guidelines for the specification of the necessary GA parameters.” This is in fact what happened, and GAs have generated a lot of interest in the field of heat transfer, in particular in the last couple of years. It is time to look back over the last 15 years or so in order to review the work accomplished with GAs in heat transfer, and then to look forward to future challenges and possibilities.

In preparing this review, we have considered major heat transfer-related journals. We have also consulted journals oriented on the numerical modeling and optimization of engineering systems for some articles that were clearly related to heat transfer, and chemical engineering journals with an important computational content. The time distribution of the reviewed publications is shown in Fig. 1. After a modest increase of the number of papers per year for the first decade, the number of publications on the topic has grown intensely starting in 2005. Even though they have some limitations, GAs appear as a promising and accessible alternative for the optimization and design of thermal systems.

The main objectives of this review could be formulated as: (i) to summarize the work related to the field of heat transfer accomplished by GAs, (ii) to compare the different GAs used so far in heat transfer, (iii) to bring out future challenges and possibilities. From the heat transfer man/woman standpoint, GAs are a tool, not a purpose in itself. Therefore, this review is not primarily concerned with the development of GAs but rather by how and when they are utilized in our field. For more details on GAs themselves, the reader is referred to specialized literature on the topic (e.g., Refs. [6,9,10,11–15]).

The review is structured as follows. A general description of GAs is provided in Section 2. Then, the reviewed articles have been grouped in different categories related to the types of problems that they addressed. The articles reviewed have been categorized as follows: design problems (Section 3), inverse heat transfer (Section 4), development of correlations and fitting (Section 5), and other applications (Section 6). Even though the separations between each family of problems are not “sharp”, this taxonomy provides a convenient way to appreciate the many problems in which GAs are used.

2. Brief description of GAs

The objective of this section is to present GAs briefly and introduce the reader to the appropriate vocabulary. A generic GA procedure will first be described, followed by a short overview of some frequent variations that can be implemented to modify the basic algorithm. This section will also mention other evolutionary algorithms analogous to GAs.

2.1. Families of GAs

Before beginning the description of the GA procedure, it is important to distinguish between single-objective and multi-objective GAs, since their algorithms are relatively different. The first one aims at finding a single set of input variables that will optimize one or many performance criteria synthesized into a single-objective function (Section 2.1.1). The purpose of the second type of GAs is to find many non-dominated solutions, also called Pareto-optimal solutions, whose performances spread over the objective functions domain (Section 2.1.2).

Another distinction has to be made between binary coding and real floating point coding [16]. In the first case, a set of input variables to optimize forms an “individual” that is represented by a binary chromosome that is then decoded into a phenotype of the real values assumed by each input variable.

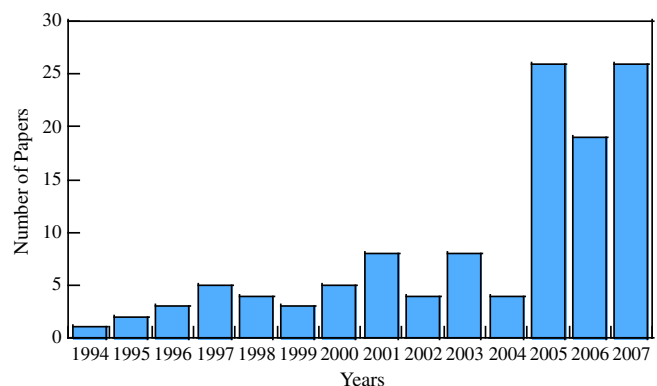
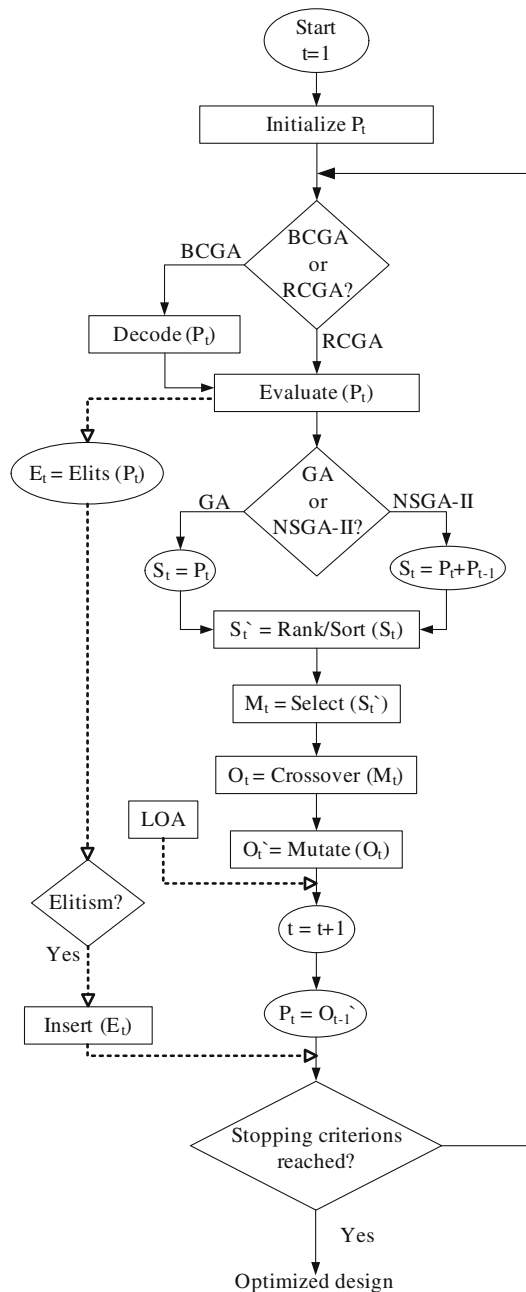


Fig. 1. The heat transfer related articles that used GAs reported in this review.



- Beginning of the algorithm
Generation $t = 1$.
- Random generation of an initial population P_t .
- If binary representation is used, then decode chromosomes into phenotypes of real values.
- Compute the objective value (s) of each individual in the population P_t .
- Keep a copy of elit individual(s) E_t .
- If NSGA-II is used, create a combined population S_t from present and previous generation individuals.
- Attribute ranking values to individuals/ Sort the population S_t .
- Select individuals that will reproduce and place them in the mating pool M_t .
- Generate the offspring population O_t by operating crossovers on M_t .
- Apply random mutations on the newly generated offsprings. Possibility of adding a local optimization algorithm (LOA).
- Index generation count.
- Form a new parent population P_t with generated offspring O_{t-1} .
- If elitism is applied, insert previous population elit (s) into the new one.
- Verify the stopping criterion. If it is not reached, restart with the new population.
- End of the algorithm.

Fig. 2. The main steps of a typical GA.

Every genetic manipulation through the algorithm is done on the binary chromosomes. For real coding, individuals are characterized only by their phenotype. In this paper, RCGA and BCGA will stand, respectively, for Real-Coded GA and Binary-Coded GA. Even though BCGAs were used more often in the reviewed papers, RCGAs experienced an increasing popularity, especially in the last few years.

2.1.1. The single-objective GA procedure

A general GA procedure is summarized in Fig. 2. Each optimization starts with a randomly generated population of individuals. Then, entering in a loop over the generations, one needs to evaluate the objective value (i.e., performance) of each individual, and attributes a fitness ranking that will drive the selection process. The evaluation of the objective value is typically the most time-consuming step of the GA procedure as it involves several simulations (one for each individual). This explains why many authors have

used an approximation of the design space associated with their problem rather than performing systematically simulations to evaluate precisely the performance of each individual. Methods such as artificial neural networks (ANN) or surface response have shown to be useful to approximate the design space in several problems (see, e.g., [17–21]).

Selection determines the individuals that will reproduce, with better chances attributed to fitter individuals. Three commonly used methods for the selection are the roulette wheel [3], the stochastic universal sampling (SUS) strategy [5], and the binary tournament [22]. These methods and others are described and compared in Refs. [23,24]. After the reproduction population is determined, a crossover operator combines couples of parents to create offspring. For BCGA, this operation is applied on the chromosomes, generally with a single or a double-point crossover method. For the RCGA, the simulated binary crossover (SBX) [25] or the blend crossover operator (BLX- α) [25] are two of the usual crossover operations performed

on the phenotypes. A low-probability mutation operator then modifies randomly some characteristics of children produced by the crossovers. In order to avoid regression in the performance through the process, many authors use elitism [3], a strategy that will be discussed later. Some authors (e.g., [26–30]) also include local search at this point of the algorithm. After verifying the stopping criterions of the loop over generations, the offspring becomes the new initial population and the process continues. A maximum number of generations and/or a maximum number of generations without improvement of the best individual generally act as stopping criterions.

2.1.2. The multi-objective GA procedure

As stated before, the multi-objective GA procedure does not search for a single optimal solution but rather for a set of solutions that represent tradeoffs between many objective functions. In most of the recent papers using this type of optimization, the specific algorithm employed is the NSGA-II [31]. A brief description of its major features follows. For a more complete view on multi-objective GAs, the reader is referred to Refs. [31–35]. The principles on which the NSGA-II relies are the same as those of the single-objective optimization: combining strongest individuals to create offspring by crossover and mutations, and repeat this scheme over many generations. However, the multi-objective optimization algorithm must take into account the fact that there are many “best solutions”, which modifies the selection process. The NSGA-II sorts the individuals based on the non-domination rank and on the crowding distance, to ensure a high level of performance as well as a good dispersion of the results. Elitism is ensured by performing the sorting process among a combined population mixing parents and offspring. The use of the binary crowded tournament for selection process allows constraint handling. This algorithm may be implemented with real-coding as well as with binary-coding.

2.2. Selection of GA parameters

Users of GAs must choose a certain number of setting (GA parameters). Unfortunately, the exact settings used for running the GAs (e.g., encoding of the design [binary form or not, precision], number of individuals in the population, mutation rate, elitist strategy and convergence criteria) are not always provided in the reviewed publications which, in some cases, might complicate the repeatability of the results or the extension of a work to a similar problem. The choices of GA parameters have a great influence on the speed of convergence as well as on the success of the optimization (i.e., finding a global optimum or the optimal Pareto front). In particular, in this review the type of representation (BCGA or RCGA), the presence or not of elitism, the size of populations, the crossover and mutation rates and the stopping criterion will receive a special attention, in order to point out efficient or less efficient parameter combinations. In the reviewed papers, mutation rate was typically lower than 5%, but exceptional cases considered much higher rates (e.g., [36]). We have found great variations in terms of the number of individuals per population, even for similar problems. Theoretical aspects describing the influence of these parameters will not be discussed, and GA settings will most of the time be presented as “numerical recipes”. For more extended studies on the optimization of the GA settings, the reader is referred to Refs. [4,37].

2.3. Frequently encountered variations

2.3.1. Elitism

Elitist strategy, first introduced by De Jong [3], is frequently implemented and is meant to eliminate regression in the perfor-

mance from one generation to the next. The most common technique used to apply elitism in single-objective algorithms consists in introducing directly one or many of the best individuals of the parent generation into the offspring generation. Another technique that is employed in multi-objective algorithms is to include the parent and offspring populations in the same mating pool in such a way that the mating population can never be weaker than the previous one.

2.3.2. Local search

Local optimization algorithms (LOA) in GAs hold their origin in the early 1990s [29,30]. Their purpose is clearly explained in [28]: “A local optimization algorithm (LOA) is often included in the GA in order to overcome such disadvantages as the inability of fine local tuning”. This operator replaces or follows the mutation operator. Local search is achieved by changing slightly one or some characteristics of promising or randomly selected individuals. However, unlike mutation, both designs (i.e., the initial and the slightly modified one) are evaluated. Generally, only the best one is kept in the population. Algorithms implementing this strategy in a GA are often called hybrid genetic algorithms (HGA).

2.3.3. Niching

Niching in GAs comes directly from ecology principles and is a general concept that can have many implications in genetic exploration [6]. The main idea behind it could be summarized as follows: when two very morphologically different individuals both perform well, we do not want to lose the specificities of each individual by mixing their chromosomes. To avoid this, a simple and common way of implementing niching is the following [38]: when two individuals competing for a place in the mating pool are too “morphologically far” (their Euclidean distance of their phenotypes is over a maximum prescribed value), the second competitor is changed for another one and so on until the two competitors are sufficiently similar. This reduces risks of eliminating very unique individuals through the tournament selection. For more information about niching, the reader is referred to Ref. [39].

2.3.4. Diversity level (DL)

The diversity level is not in itself a strategy, but a criterion that can drive the crossover and mutation rates. It is defined as the ratio between the best fitness and the average fitness of a population [40–41]. Briefly, the higher is DL, the more crossovers are important compared with mutations and vice-versa. This criterion helps keeping a good diversity of individuals throughout the algorithm.

2.3.5. Micro GA (μ GA)

Micro-GA [42] is a quick alternative to standard GA when computational time is an issue. It generally uses less than 10 individuals per population and basically applies the same scheme as standard GA, except that it does not use the mutation operator. However, the fact that the population is small leads faster to local optima. To preserve diversity, when convergence is declared, the algorithm restarts with a new random populations, in which is inserted only the best individual found before. This forms a loop of short GA procedures, until the total generation limit is reached. This algorithm leads to a smaller number of individuals to evaluate than standard GA and is thus faster.

2.3.6. Other evolutive algorithms

As this review concentrates on GAs, we will only mention here some of the other evolutive strategies that appear in the papers that will be reviewed. Differential evolution (DE), introduced by Storn and Price [43], is a non-binary method featuring a crossover method relying on the use of weighted differential vectors between individuals. Its multi-objective version, the multi-objective differ-

ential evolution (MODE), is described in [44]. Another popular algorithm is the simulated annealing (SA) [45] which, as its name suggests, relies on the physical phenomenon of the improvement of properties by annealing.

2.4. Advantages and drawbacks

One of the main advantages of GAs as opposed to other optimization techniques is their ease of use. Some free routines are available on the web (e.g., [46]). Furthermore, easy-to-use commercial GA toolboxes are now available, e.g., [47]. One of the specificities of GAs is that they do not necessitate the calculation of the objective function gradient with respect to the design variables. This feature is particularly helpful in some cases such as for material allocation, ordering (combinatory) problems, multi-objective problems, MINLP (mixed integer non-linear programming).

GAs are also recognized for their robustness. Due to their probabilistic nature, the initial guess has a low incidence on the final result of the optimization. When appropriately handled, GAs will explore a large portion of the design space, and are unlikely to converge to a local optimum. GAs “search from a population and not from a single parameter set. They are capable of searching for solutions from disjointed feasible domains. They can operate on irregular functions and those that are not differentiable. They do not require the computations of gradients” [48].

It is also interesting that after convergence is declared, the user ends up with a population of individuals, many of which are likely to provide alternatives to the “best” individual of the population. GAs produce not just one optimal individual, but a population of good individuals. From a design point-of-view, it is useful to have available a collection of possibilities to choose from. Moreover, this feature of GAs allows, for example, to perform a pre-optimization to identify potentially good designs with a simplified model, and then to evaluate more precisely with a better model a certain number of the best individuals produced by GAs [49].

Finally, GAs are easily parallelized. In fact, the objective function of several individuals of a population could be calculated simultaneously on different processors.

One disadvantage of GAs is that they can be slow to converge compared to other optimization methods that have been developed specifically for a problem. This is especially the case for larger and hard-to-solve problems such as topology optimization or inverse problems, as we will see below (see, e.g., [50,51]). GAs may also be less efficient and slower than traditional methods when dealing with very simple problems, especially those for which an analytical optimum is known. Finally, it is important to note that several runs of a GA with the exact same settings could potentially converge to different nearly-optimal results because of their probabilistic nature. Therefore, repeatability is not always perfect.

3. Design of thermal systems

The first family of heat transfer problems addressed with GAs is the design of thermal systems. Design is considered here with a broad sense: shaping, sizing, placing, ordering, etc. The thermal systems that have been considered so far in literature are many: heat exchangers, heat and fluid flow networks, fin, porous media, heat sinks, etc. This first family of heat transfer problems in which GAs are used is by far the one that counts the most published papers ($98/132 = 74\%$).

As we will see below, the “complexity” involved in the modeling of these systems varies greatly, ranging from simple analytical equations to advanced CFD. Tables 1–8 present a schematic summary of the most important features of the publications on design problems. For each paper, the objective function(s), the level of modeling (e.g., analytical expression, CFD, etc.), and the number of design variables are reported in the appropriate table. Details related to the GA are also presented in the tables: single vs. multi-objective (S/M), binary vs. real-coded (B/R), crossover rate (Pcross), mutation rate (Pmut), number of individuals per population (Nind), maximal number of generations (Maxgen) or other convergence criterion, and the presence or absence of an elitist strategy (Y/N).

3.1. Optimization of systems producing, transferring and converting energy

We present in this sub-section recent work related to optimization of systems producing, transferring, and converting energy, and in particular, thermal energy. One common denominator of these articles is that the modeling on which they rely is most of the times based on algebraic equations and correlations. Nevertheless, the number of possible designs is typically quite large, and GAs are helpful to determine the best options.

3.1.1. Heat exchangers

Heat exchangers are an integral component of all thermal systems. Their designs should be adapted well to the applications in which they are used; otherwise their performances will be deceiving and their costs excessive. Heat exchanger design can be a complex task, and advanced optimization tools are useful to identify the best and cheapest heat exchanger for a specific duty. GAs are among the current options to perform such work, see Table 1. The models used to evaluate the performance of HES are mostly analytical and rely on empirical relations.

Selbas et al. [52] designed shell-and-tube heat exchangers with a standard BCGA without elitism. The objective was to minimize the cost, based on tube diameter, tube pitch, number of passes, shell outer diameter and baffle cut. Wildi-Tremblay and Gosselin [53] also used a GA to minimize the cost of shell-and-tube heat exchangers for a specified duty. The cost included

Table 1
Summary of the GAs used for the design of heat exchangers.

Article	Problem			GA						
	Objective	Model	No. of variables	Single/multi	Bin/real	Pcross	Pmut	Nind	Maxgen	Elitism
[52]	Min. cost	Analytical	6	S	B	N/A	N/A	N/A	100	N
[53]	Min. cost	Analytical	11	S	B	0.7	0.04	30	300 w/out impr.	Y
[54]	Min. cost	Analytical	11	S	B	0.7	0.04	30	300 w/out impr.	Y
[55]	Min. cost	Analytical	7	S	R	N/A	N/A	40–100	15	N/A
[56]	Min. cost	Analytical	3	S	B	0.5	N/A	20	100	Y
[57]	Min. cost	Analytical	8	S	R	0.4	0.05	80	50 w/out impr.	Y
[58]	Max. heat transfer per pumping power	Analytical	4	S	R	N/A	0.05	6000	30	Y
[17]	Min. cost or weight	Analytical	5	S	N/A	N/A	N/A	N/A	N/A	N/A
[59]	Min. cost or volume	Analytical	3	S	N/A	N/A	N/A	N/A	1000	N/A
[48]	Max. NTU per unit of pressure drop	Analytical	3	S	R	0.7	0.01	150, 250	N/A	N/A

Table 2
Summary of the GAs used for the design of heat exchanger networks (HENs), chemical plants, and design integration.

Article	Problem			GA						
	Objective	Model	No. of variables	Single/multi	Bin/real	Pcross	Pmut	Nind	Maxgen	Elitism
[60]	Min. cost	Analytical	27	S	R	N/A	N/A	20	100	Y
[61]	Min. cost	Analytical	~68	S	N/A	N/A	N/A	20	10	N/A
[62]	Min. cost	Analytical	N/A	S	R	0.5	0.3	50–100	150–200	N/A
[36]	Min. cost	Analytical	(1) 1–6 (2) 1–17	S	R	N/A	(1) 0.4 (2) 0.1–0.8	(1) 50 (2) 20	(1) 57 (2) 8	N/A
[63]	Min. required utilities	Analytical	N/A	S	R	Variable	Variable	100	500–1000	N/A
[64]	Min. cost	Analytical	7–20	S	R	N/A	N/A	N/A	N/A	N/A
[65]	Min. cost	Analytical	1600	N/A	N/A	N/A	N/A	N/A	N/A	N/A
[18]	(1) Max. production (2) Max. selectivity (3) Min. toluene-benzene use	Ordinary differential and non-linear algebraic eqs.	Up to 13	M	R	0.7	0.05	80	100	Y
[66]	(1) Max. production (2) Max. selectivity (3) Max. yields	Ordinary differential and non-linear algebraic eqs.	4	M	R	0.9–0.7	N/A	100–150	600	N/A
[67]	(1) Max. production (2) Max. selectivity (3) Max. yields	Ordinary differential and non-linear algebraic eqs.	4	M	B	0.5–0.7	0.002	50	100	N
[68]	Max. yields of reaction	Fuzzy neural network experimental based	3	S	R	0.5	0.0–0.1	120	100	N/A

the cost of purchase and the cost of operation. Eleven design variables were considered (tube pitch, tube layout pattern, number of tube passes, baffle spacing at the center, baffle spacing at the inlet and outlet, baffle cut, tube-to-baffle diametrical clearance, tube bundle outer diameter, shell diameter and tube outer diameter), and each design was represented by a string of 24 bits. Correlations were used to estimate the heat transfer coefficients and pressure drops. The GA identified the optimal design approximately 22 times faster than the testing of all possibilities. Allen

and Gosselin [54] extended this work to condenser shell-and-tube. An additional design variable was the side (shell or tube) where the condensing fluid flows. The designs were represented with 27 bits. In Ref. [55], Babu and Munawar minimized the cost of shell-and-tube heat exchangers based on the differential evolution (DE) optimization method. Seven design variables were taken into account. Caputo et al. [56] also minimized the cost of shell-and-tube heat exchanger, but only three design variables were included (shell diameter, tube diameter, baffle spacing). The GA in-

Table 3
Summary of the GAs used for the design of heating, ventilating, air conditioning and refrigerating (HVAC&R) systems.

Article	Problem			GA						
	Objective	Model	No. of variables	Single/multi	Bin/real	Pcross	Pmut	Nind	Maxgen	Elitism
[69,40]	Min. power required	Analytical	N/A	S	B	Var.	Var.	N/A	N/A	Y
[70]	(1) Min. energy consum. (2) Min.% dissatisfied	Analytical	Up to 73	M	R	0.9	0.04	100	500	Y
[71]	Min. energy consumed	Analytical	Max 15	S	B	0.7	0.01	100	500	Y
[72]	Min. energy consumed	Analytical	9	S	N/A	N/A	N/A	N/A	N/A	N/A
[73]	(1) Min. operating costs (2) Min. discomfort	Analytical	200	M	B	N/A	N/A	200	1000	N
[74]	Min. err. metrics on $T(t)$	Analytical	2	S	B	0.85	0.02	40	50	Y
[75]	Min. err. on T	Analytical	2	S	B	N/A	N/A	30	N/A	N/A
[76]	Min. cost	Fuzzy logic, experimental based	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
[77]	Min. err. T_{set} vs. T_{actual}	Analytical	2	S	B	N/A	N/A	20	N/A	N/A
[41]	Min. power required	Analytical	N/A	S	B	Var.	Var.	50	100	Y
[78]	Min. power required	Analytical	4	S	B	0.6	0.01	100	100	N/A
[79]	Min. energy consum. + Min. over-dimension.	Analytical	6	S	B	0.6	0.01	100	100	N/A
[80]	Min. power consum.	Analytical	4	S	B	0.8	0.01	50	100	N/A
[81]	Min. power consum.	Analytical	4	S	N/A	N/A	N/A	1	500–800	N/A
[82]	Min. power consum.	Finite elements	5	S	N/A	N/A	N/A	N/A	N/A	N/A
[83,84]	Max. COP	Analytical	9	S	B	100%	0.01	100	50	N/A
[19]	Min. cost	ANN experimental	4	S	B	0.5	0.033	50	100	N/A
[85]	Min. cost	CHEOPS tool	12	S	R	N/A	N/A	200–1000	N/A	Y
[86]	Min. life-cycle cost and environmental impact	Analytical	8	M	N/A	0.9	0.02	40	200	Y
[87]	Min. cost	Analytical	33	S	N/A	N/A	N/A	N/A	4000	N/A
[88]	Min. cost	Analytical	8–16	S	R	N/A	0.01–0.2	N/A	200–1000	Y

Table 4

Summary of the GAs used for the design of power generation systems.

Article	Problem			GA						
	Objective	Model	No. of variables	Single/multi	Bin/real	Pcross	Pmut	Nind	Maxgen	Elitism
[89]	(1) Min. η_p, η_t ST (2) Max. TSFC	Isoentropic analytical model for turbojet	2	M	N/A	0.8	0.02	100 and 200	N/A	Y
[20]	(1) Max. efficiency (2) Min. NO _x emission	ANN on experimental data	10	M	N/A	N/A	N/A	N/A	1000	N/A
[90]	Max. efficiency	2-zones combustion model	6	S	N/A	N/A	N/A	N/A	N/A	N/A
[91]	Min. cost	Algebraic	N/A	S	Integers	0.8	0.005	10	1000	Y
[92]	Max. net present value	Algebraic	9	S	Mixed	N/A	N/A	N/A	N/A	N/A
[93]	(1) Min. cost (2) Max. cash flow	Algebraic thermodynamic	From 4 to 12	S	N/A	N/A	N/A	N/A	N/A	N/A
[94]	Max efficiency	Algebraic thermodynamic	6	S	B	0.8–0.9	0.02–0.04	50	300	Y
[96]	Min. cost	Hourly annual simulation	3	M	N/A	0.33	0.8	500	600	Y
[97]	(1) Min. cost, (2) Min. demand-vs-supply error, (3) Min. gas discharge	Algebraic	N/A	M	N/A	N/A	0.04	N/A	50	Y
[98]	Min. cost	Algebraic energy balance	S	B	N/A	0.04	N/A	100	Y	
[99]	Max. production time	N/A	N/A	S		0.8	0.1	30	400	Y
[100]	Min. cost	Algebraic	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
[101]	Min. cost	Algebraic	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
[102]	Min. cost	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
[103]	(1) Min. cost (2) Max. exergy efficiency	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
[104]	Max. power	Algebraic energy balance	7	S	B	1	0.01–0.06	10–100	100	Y
[105]	Min. cost	Algebraic energy balance	N/A	S	N/A	N/A	N/A	N/A	N/A	N/A
[106]	Min. cost	Algebraic energy balance	N/A	S	B	0.9	0.9	5500	20	N/A
[56]	Max. COP	Heat exchanger energy conservation (coupled EDO)	N/A	S	B	0.7	0.02	1000	20	N
[108]	Max. COP	Algebraic equation	3	S	R	N/A	N/A	N/A	60	N/A
[95]	Min. cost of expansion	Analytical	28	S	B	N/A	0.1–0.001	50–150	1000	N/A
[109]	Max. cooling capacity	Algebraic equation	3	S	R	0.6	0.2	20	200	Y

cluded 20 individuals per population, elitism, and a scattered crossover method where a random binary vector is created having a number of bits equal to the number of genes of an individual. Then, the genes where the value is 1 are copied from the first parent, while the genes where the value is 0 are copied from the second parent. A single-objective function representing the annual cost (exergetic and capital costs) of a shell-and-tube heat exchanger was minimized in [57]. The function depends on tube length (discrete), outer diameter of the tubes (discrete), pitch type (discrete), pitch ratio (discrete), tube layout angle (discrete), number of tube passes (discrete), baffle spacing ratio (discrete) and mass flowrate of the utility (continue). A mixed discrete and continue real value GA was used. Original features were added to the GA such as the insertion of new randomly generated individuals in each generation.

The heat transfer per unit of pumping power was maximized for a corrugated sandwich panel with an algebraic model by Valdevit et al. [58]. The design variables were the core thickness, the web thickness, the angle of corrugation and the order of corrugation. The population counted 6000 individuals. An elitist strategy was implemented. The GA was run for 30 generations. In [17], the morphology of a plate fin heat exchanger was optimized based on two objective functions that were treated separately: the weight and the operation cost. The length, width, number of hot side layers, fin height at hot/cold side, fin pitch at hot/cold side were the design variables. Constraint handling was operated by a back propagation artificial neural network (ANN) that was used to eliminate individuals who violate the constraints before they were actually calculated by the GA. The training of the ANN was made with the first GA population of 500 individuals.

Table 5

Summary of the GAs used for the design of fins.

Article	Problem			GA						
	Objective	Model	No. of variables	Single/multi	Bin/real	Pcross	Pmut	Nind	Maxgen	Elitism
[110]	Max. fin effectiveness	2D conduction	5	S	R	0.2	N/A	100	N/A	N
[111]	Uniformize heat flux distribution	1D–2D conduction	12	S	B	N/A	N/A	99	25	N/A
[112]	Max. heat transfer rate per unit of mass or min. entropy generation	Correlations from numerical simulations with conduction equation	5	S	B	0.6	0.1	50	50	N
[113]	Min. thermal resistance	1D–2D fin equation	3	S	N/A	N/A	N/A	N/A	N/A	N/A
[114]	Max. Nusselt number or heat flux per unit of length or surface	2D conduction for T and 2D diffusion for u	Up to 8	S	N/A	N/A	N/A	20	40	N/A
[115]	Max. Nusselt number or heat flux per unit of length or surface	2D conduction for T and 2D diffusion for u	Up to 8	S	N/A	N/A	N/A	20	50	N/A
[116]	Max. Nusselt number or heat flux per unit of length or surface	2D conduction for T and 2D diffusion for u	Up to 8	S	N/A	N/A	N/A	20	N/A	N/A
[117]	Max. Nusselt number or heat flux per unit of length or surface	2D conduction for T and 2D diffusion for u	Up to 4	S	N/A	N/A	N/A	20	N/A	N/A

Table 6
Summary of the GAs used for the other examples of design problems based on conduction equation.

Article	Problem			GA							Elitism
	Objective	Model	No. of variables	Single/multi	Bin/real	Pcross	Pmut	Nind	Maxgen		
[50]	Min. hot spot T	2D cond. volume-to-point method	44	S	R	N/A	N/A	N/A	N/A	Y	
[119]	Min. stress	2D cond. Element-free Galerkin method.	8	S	R	Var	Var	80	10gen with 0.1% improve	Y	
[120]	Min. err. on T dist.	2D point heat sink method	3	S	R	0.6	0.002	50	N/A	Y	
[121]	Min. heat loss	2D cond.	4	S	B	0.5	0.15	30	100	N/A	
[122]	Max. heat transfer	2D cond.	14	S	R	1	1/14	20	200	Y	
[118]	Min. hot spot temperature	2D cond.	4	S	N/A	0.8	N/A	100	200	Y	

Table 7
Summary of the GAs used for the design of thermofluid systems.

Article	Problem			GA							Elitism
	Objective	Model	No. of variables	Single/multi	Bin/real	Pcross	Pmut	Nind	Maxgen		
[123]	Max. Nu, min. friction	2D CFD (4 eqs.)	5–11	M	Discrete	N/A	N/A	20–50	30	Y	
[124]	Min. temperature	2D heat transfer	4–10	S	B	0.7	0.04	30	200 w/out impr.	Y	
[26]	Min. temperature	2D heat transfer	10	S	B	N/A	N/A	25	250 w/out impr.	Y	
[27]	Min. temperature	2D heat transfer	60	S	B	N/A	0.05	20	300 w/out impr.	Y	
[125]	Min. temperature	2D heat transfer 2D Darcy flow	14	S	B	0.8	0.04	30	200 w/out impr.	Y	
[126]	Max. conductance	2D CFD	Up to 41	S	N/A	N/A	N/A	100	20 with less than 1% diff.	N/A	
[127]	Max. global conductance	2D CFD	Up to 3	S	N/A	N/A	No	5	N/A	Y	
[128]	Max. heat transfer min. pressure drop	CFD	4	M	Real	0.167	variable	30	20	Y	
[8]	Min. failure rate	2D CFD	7	S	Integer	0.6	0.1	7	7	Y	
[130]	Min. pressure drop max. heat transfer	CFD	Approx. 20	M	B	0.9	0.05	100	500	Y	
[131]	Max. heat transfer	2D CFD laminar	4	S	N/A	N/A	N/A	12	N/A	N/A	
[112]	Min. entropy generation max. heat transfer	1D conduction in fins 1D energy balance in fluid + correlations	5	S	B	0.6	0.1	50	50	N/A	
[132]	Min. entropy generation	2D CFD	4	S	B	0.7	0.02	100	500	N/A	
[133]	Max. effectiveness	CFD	8	S	N/A	N/A	N/A	N/A	N/A	N/A	
[134]	Min. warpage	Polynomial best fit based on several CFD results	3	S	16	1	0.1	50	500	N/A	
[135]	Max. heat transfer max. flow rate	2D CFD	6	M	N/A	N/A	N/A	15	30	N/A	
[59]	Min. cost min. volume	Correlations	3	S	B	0.5	0.005	50	1000	Y	

Table 8
Summary of the GAs used for the design of radiation dominated system.

Article	Problem			GA							Elitism
	Objective	Model	No. of variables	Single/multi	Bin/real	Pcross	Pmut	Nind	Maxgen		
[136]	Min. residuals between desired and estimated values of heat flux	Radiative transfer eq. with discrete transfer method	Up to 11	S	B	N/A	N/A	N/A	100	Y	

In [59], based on correlations for heat exchange and pressure drop, Xie et al. optimized a plate fin compact heat exchanger. The objective was to minimize either the cost or the volume under constraints (maximal allowable pressure drop and minimum efficiency requirement). Three shape parameters of the heat exchangers were varied for 1000 generations. The NTU per unit of pressure drop of a direct-transfer type intercooler and a direct-transfer type regenerator was maximized by a real-coded GA, based on correlations in [48]. Three design variables (x , y , z dimensions) were present. For the first case considered, the size of the population was 150, the crossover and mutation probabilities were 70% and 1%,

and the tournament probability and scale for mutation were also 70% and 1%. For the second case, the population was larger (250).

3.1.2. Heat exchanger networks (HENs), chemical plants, and design integration

In addition to the detailed design of the heat exchangers themselves as described in Section 3.1.1, genetic algorithms have been considered for designing heat exchanger networks (HENs) and facilitate heat integration in many applications (Table 2). Typical objectives of these design problems are to maximize the energy recovery or to minimize the cost of the design.

Pettersson and Soderman [60] designed a heat recovery system that minimizes the total cost, which is a function of the heat exchanger areas, number of heat exchangers, and cold and hot utility costs. The design variables were the existence/non-existence of fluid matches (binary variables) and the area of the heat exchangers. Roulette wheel selection was considered for the selection, and a two-point crossover operator was considered. When designing retrofit large HENs, GAs can help separating a large system into smaller subsystems, as in Ref. [61]. For a fixed hot/cold fluid network topology, [62] minimized the cost of structural change (first step), and the cost of exchangers parameters changes (second step) with a GA. In the first step, 100 individuals formed the population, and evolved for 200 generations. In the second step, 50 individuals evolved for 150 generations. The authors reported computational times between 2 and 12 h. Two steps of the HEN optimization relied on GAs in Ref. [36]: (i) the optimization of ΔT_{\min} was performed to minimize the cost of capital and energy, and (ii) the optimization of the network was achieved by minimizing the total cost for a certain value of ΔT_{\min} . Mutation rate was variable, but generally very high (up to 80% in some cases). In [63], the HEN was optimized to minimize the additional utilities required to reach a fixed performance criterion. The morphology of the network for five heat exchanger layers was optimized by a binary GA. The mutation rate was large (10%) during the first 500 generations and smaller (1%) afterwards. Ma et al. [64] minimized the total exergetic and capital cost (based on heat transfer surface) of a multi-period heat exchanger network with a real value GA. The positioning of the heat exchangers in the networks and the temperature difference were the design variables. Design integration of multi-period process system was performed in [65] by minimizing operating and capital annualized costs with a GA.

In addition to heat exchanger networks, many authors optimized chemical reactors and plants. For example, optimization of styrene reactor and process with MODE, NSGA and NSGA-II is addressed in Refs. [18,66,67]. Three objectives were pursued simultaneously: maximize productivity, selectivity and either maximize yields or minimize heat duty. In [66], a multi-objective differential evolution (MODE) approach was compared to the binary niching adapted NSGA proposed in [67] to solve the same problem and converged to a better Pareto front. Tarafder et al. [18] used real-coded NSGA-II with simulated binary crossover (SBX) and successfully obtained the optimal Pareto front. Ref. [68] optimized the feedstock composition and operating conditions for a secondary reaction of FCC gasoline. Experimental data were fed to a neural network with fuzzy logic to evaluate the yields of the reaction. The GA used was real-coded, with $N_{\text{ind}} = 120$, $P_{\text{cross}} = 50\%$, $P_{\text{mut}} = 0\text{--}10\%$. A maximum of 100 generations were calculated.

3.1.3. Heating, ventilating, air conditioning and refrigerating (HVAC&R) systems

The main issue addressed so far with GAs for HVAC and refrigeration systems is concerned with the design of the system itself, or with the control of the systems for minimizing cost, minimizing energy consumption and maximizing comfort. The evaluation of the system performances is based mainly on algebraic equations. This body of work is abundant and still growing, see Table 3.

For example, Lu et al. considered a GA to minimize the total power required for an HVAC system [40,69]. Several design variables were taken into account: number of operating chillers, number of operating chilled water pumps, number of operating cooling coils, number of conditioned rooms, number of operating condenser water pumps, number of operating cooling tower fans, temperature of chilled water supply, temperature of condenser water supply, air flow rate of supply air through k th cooling coil to l th room, and condenser water flow rate provided by m th pump. Simulations were performed with 60 rooms, 15 cooling coils, 3 water

pumps, 3 chillers, 3 condensed water pumps, and 3 cooling towers. The optimal sequencing on a 24 h test period resulted in a 800 kWh save compared with traditional control methods. Optimal pressure set points of pumps were found by an adaptive neural fuzzy inference system (ANFIS) and the calculation of the objective function was made by an analytical model. To ensure diversity, the probability of crossovers and mutations were functions of the diversity level (DL), which is the ratio between the best fitness and the average fitness of a population. In [41], the power consumption of chillers, pumps and fans in an HVAC system was also minimized by a binary elitist GA. The objective values were calculated from an adaptive neuro-fuzzy inference system (ANFIS). The design variables were the number of chillers, of water pumps and of fans, the temperature of the chilled water supply, and the air-flow rate of supply to the k th cooling coil in the i th room. The crossover and mutation probability were defined by the diversity level (DL).

Predicted percentage of dissatisfaction (comfort zone) and energy consumption were minimized simultaneously by a GA in the real-time computation of the setting of an HVAC system in [70]. The optimal parameters that were determined by the GA were the duct static pressure set point, the supply air temperature set point, the chiller water temperature, the N zones temperature set points, and the required reheat. NSGA-II was used with real-coding. The population size was 100. In Ref. [71], the power consumed by the machinery of an HVAC system (i.e., pumps, chillers and cooling towers) has been minimized. The GA determined which of many HVAC systems put in parallel were in use with a binary variable (yes/no) associated with each component. Water and air flows were also optimized with a precision of 8 bits each. One hundred individuals evolved for a fixed number of generations (500). In [72], the on-line control of outdoor air (outdoor air ratio, voltage of reheaters) for air conditioning system has been optimized with a GA to minimize the energy consumed.

A multi-objective GA was put to use by Wright et al. for building optimization by a three-day hour-by-hour simulation [73]. The objectives were to minimize the operating cost and the thermal discomfort. Constraints were handled by introducing the infeasibility criterion (global constraints violation measure) as a third objective. The design variables were: on/off status for 15 h/day (unoccupied period), supply air flow rate and temperature for each hour, coil width, coil height, number of rows and water circuits in each of the two coils, water flow rate, fan diameter and heat recovery device size, which totalized 200 design variables over the 3 days. 200 individuals evolved for 1000 generations.

The optimal control of HVAC systems with the help of GAs has been addressed in Refs. [74–75]. Huang and Lam [74] optimized the proportional and integral parameters of a PID controller for regulation of an HVAC system. A single-objective function combining overshoot measure, settling time and mean square error was employed. A standard elitist BCGA was considered with a population of 40 individuals and a maximal number of generations of 50. Each controller design was represented by a string of 16 bits. In [75], authors designed a PID controller for a variable air volume (VAV) AC system. Two controller parameters were chosen by the GA to minimize the error between commanded and measured temperature in a test-room. A GA determined the fuzzy logic parameters for a self-adapting building to minimize energy consumption regarding occupancy in [76]. The control of a supply air temperature with variable air volume systems was assured by a BCGA in [77]. This was achieved by minimizing the difference between a set-point for the air temperature and the actual temperature. Two control parameters totalizing 20 bits were determined by the GA. Convergence was declared when the average performance of the population was similar to the performance of the best individual of the population.

The part load ratios of 4 chillers have been determined by a BCGA in [78] to minimize the power consumption. A penalty was added when the cooling capacity was not reached. The population was made of 100 individuals, each of 40 bits. The roulette method was used for the selection. The part load ratio of each of the 6 chillers of a system has been optimized by a GA in [79]. The objective was to minimize the energy consumption for fixed cooling loads and minimize the discrepancy between needed and produced cooling (i.e., minimize over-dimensioning). One hundred binary-coded individuals evolved for 100 generations. The mutation rate was 1%. Chang [80] found up to 4 chilled water supply temperatures of chillers when solving the chiller loading problem (minimum power consumption) with a GA. Ten bits per temperature were used. The population size was 50 and the number of generations was fixed to 100, with crossover and mutation probabilities of 80% and 1%. The author notes that “after analysis and comparison of the case study, it has been concluded that [the GA] not only solves the problem of Lagrangian method, but also produces results with high accuracy within a rapid timeframe.” This chiller loading problem had been addressed before with an evolutionary strategy (ES) in which a single design evolved thanks to mutations [81]. When the mutation was beneficial, the new design would replace the previous one (random search). Refs. [80,81] showed very similar results about the accuracy of both evolutionary methods and on the computational power savings realized when using it instead of the Lagrangian method. Thus no sensible differences in the optimization results between ES and GA can be reported in this case. A GA found up to 5 control parameters of a heat dissipater crossed by air [82]. Important fan power consumption reductions were achieved thanks to the optimization of the control system. In [83–84], multistage cooling cycles have been designed with a binary GA with imposed temperatures and heat removing rates. The objective was to maximize the multistage effectiveness (COP of multistage/COP if many single stage cycles). The physical configuration of the system (8 bits), the pressure range of the system (10 bits) and the split ratios (20 bits) were the design variables. One hundred individuals constituted the population. To build the mating pool, the fitness value of each individual has been normalized by the average population fitness. A random number between 0 and 1 was added to this normalized fitness and the result gives the number of copies of each individual in the mating pool. The pool filled up until it reached the population size. Uniform, single and double crossovers were performed. The mutation rate was 1%. [19] optimized direct-fired absorption chillers. The operation cost (costs for chillers, pumps and fuel) was minimized under different cooling loads by varying the chilled and cooling water mass flow rate, the chilled water supply temperature, and the cooling water return temperature. The GA used was a standard BCGA with $N_{ind} = 50$, $P_{cross} = 50$, $P_{mut} = 0.033$ and $Max_{gen} = 100$. An artificial neural network trained with 2100 sets of experimental data was used to evaluate the individuals.

Two different problems of optimization of building parameters (geometry of the building, composition of walls and floors, and solar protection) regarding heating and cooling loads were addressed separately by Znouda et al. [85]: minimize the power consumption and minimize the cost. The GA used a standard elitist strategy with real coding. Wang et al. [86] proposed a green building design method based on a multi-objective GA. The building design parameters considered were the building orientation, the building aspect ratio, the type of windows, the window-to-wall ratio, the wall type, the wall materials, the roof type and the roof materials. The two objectives considered were the life-cycle cost and the life-cycle environmental impact.

Curti et al. [87] used an analytical model to represent a district heating network. A GA minimized the total cost of the network by adjusting 33 design variables related mainly to the operating tem-

peratures and mass flow rates, but few information about the GA was available. Chan et al. [88] optimized with a RCGA a pipe network to minimize the pumping power and installation cost (combined into a global cost) of a district heating network. The network was characterized by the link between the stations, which were between 9 and 17. The GA was hybridized with local search. Forty percent of the generated individuals of a population came from crossovers, the other 60% being mutated individuals of the previous generation.

3.1.4. Power generation

The generation of power is another body of work that is very abundant, and in which GAs can play a significant work to maximize performance and minimize cost (Table 4). The topics covered include engines, fuel cells, PVs, hydro-thermal plants, combustion, etc. In particular, complex power generation systems including several components and several energy sources have proved to benefit from GAs to achieve good tradeoffs between competing objectives.

For example, a multi-objective GA was considered to optimize a subsonic turbojet engine based on an analytical model in [89]. The thermal efficiency, the propulsive efficiency, the thrust-specific fuel consumption and the specific thrust were the four objectives. Up to three design variables were chosen by the GA: input flight Mach number, compressor pressure ratio and turbine inlet temperature. A population size of 200 has been chosen with crossover probability and mutation probability of 0.8 and 0.02, respectively. A coal burner is designed with a GA in [20] to maximize efficiency and minimize NO_x emission. These objectives were calculated from an artificial neural network (ANN) trained with experimental results. A comparison study between GA, artificial neural networks and fuzzy logic is proposed in [90], in order to maximize the efficiency of an engine, under emission constraints. The design variables were the equivalence ratio, the charge pressure, the charge temperature, the combustion duration, the combustion start and the form factor. The ANN proved to perform better than the GA, but no information on the GA was given.

The overall cost (investment and operation costs) of a power plant expansion was minimized by Sirikum and Techanitisawad [91], under power and environmental constraints. The GA determined the year of implementation of each energy generating unit. The population was composed of 10 individuals, which evolved for 1000 generations. The phenotypes were integers. [92] maximized the value (based on economic, thermal efficiency and emissions) of a cooling, heating and power generation system. The variables considered included the gas turbine series and number, the gas engine series and number, the gas turbine side hot water mass flow share, the gas turbine side cold water mass flow share, the hot water supply temperature, the pinch floor cooling device and the cold water supply temperature. This work was performed with a GA with mixed integer and real value coding. Ref. [93] optimized gas turbine power plants for single, dual and triple pressure with and without reheating. Two objectives were addressed separately, i.e., minimize the energy generation cost and maximize the cash-flow. Pressures and temperatures in the cycles were varied by the GAs. In [94], the internal efficiency of a steam turbine has been maximized by a BCGA. Velocity and angle of flow on different parts of the turbine were the design variables. The GA included 50 individuals per population, with probability of crossover and mutation, respectively, between 80% and 90%, and 2% and 4%. In [95], a standard GA and a variant in which mutations are replaced by simulated annealing (ASAGA) were compared for the expansion planning problem. ASAGA converged faster and provided a 3% lower cost than the standard GA. Different combinations of population size, mutation rate, and crossover approaches were studied.

A GA (NSGA-II) optimized a photovoltaic (PV) – wind hybrid system in [96]. Three objectives have been considered simultaneously: the total system cost, the autonomy level and the wasted energy rate. Hour-per-hour simulations with weather data were performed to evaluate these objectives. The GA determined the PV array peak power, the wind generator rated power and the rated capacitors of the battery. Ranking based on domination rank and crowding distance, and tournament selection were exploited by the GA. A BCGA optimized the operation of an energy system that uses in combination a solar power module, proton-exchange membrane fuel cell cogeneration (PEMFC-CGS) with methanol steam reforming, a geo-thermal heat pump, heat storage and battery, commercial power, and a kerosene boiler in [97]. Three objectives (minimization of operation cost; minimization of the error of demand-and-supply balance; and minimization of the amount of greenhouse gas discharge) were grouped into a single weighted objective function. The GA obtained the electric power output, heat output, power and heat to be stored at each time step and for each piece of equipment. The population evolved for 50 generations, with elitism, and a mutation rate of 4%. In [98], a BCGA was used for optimizing operation and planning of multi-device energy supplier. The objective was to minimize the operation cost.

Oldenburg et al. [99] optimized the schedule process assignation of multi-production plants with a BCGA, in order to minimize the energy cost. The GA had to decide to which reactor each process went, and the schedule of the processes. Each population was made of 30 individuals. The crossover probability was 0.8 and a two-point crossover was implemented. The mutation rate was 10%. The scheduling of hydro-thermal power generation to minimize production cost has been performed by different optimization methods (i.e., Swarm Optimization (PSO) and cultural algorithm (CA)), including GAs, in Refs. [100,101]. Ref. [102] reviews a cogeneration plant optimization realized with GAs which resulted in annual savings of 0.3 M\$. A general procedure based on an extended hybridized GA called the innovative multi-objective optimization (IMOO) is developed in [103] to solve multi-objective (such as maximizing the exergy efficiency and minimizing the operation cost) optimization problems of large scale industrial production systems. In the cogeneration industrial process presented as an illustrative example, the design variables were the compressor pressure ratio, compressor isentropic efficiency, the gas expander isentropic efficiency and the temperature at two different points of the thermodynamic cycle. To avoid the emergence of superdominant individuals or local optimum, non-dominated individuals (elits) of each parent population were removed and archived in a bounded-size database. To control the size of this archive, the selection was made by a crowded method, which ensured diversity among archived elits. Then, the mating pool was constituted from the elit-free parent population and the selected archived elits. So, there could not be double representation of a dominant design because it was eliminated by the crowded selection operator of the archive. A standard binary GA maximized the total electrical power generated in a cogeneration system with steam boilers and steam turbine generators [104]. Steam flow rates in 7 points of the network were optimized.

Obara minimized the installation and operation cost of fuel cell energy supplier [105]. The GA found the fuel cell position, reformer position, hot water piping path and gas production of reformer. The model included mass and energy balances. In [106], fuel cell equipment characteristics were determined by a BCGA in order to minimize cost. To assess the performance of the fuel cell, energy and mass balance for the fuel cell, gas pipes and electric line network were solved. The population of designs evolved for only 20 generations.

In [107], the thermoelectric cooler performance was also maximized, but with a non-elitist BCGA. The electric current and cur-

rent distribution were varied. The population was large (1000). The crossover and mutation rates were 70% and 2%, respectively. Authors also used simulated annealing (SA) and obtained similar results, but in a shorter computation time and with less effort. Some years later, thermoelectric coolers (TEC) were optimized by RCGAs in Refs. [108,109]. The objective in each case was to maximize the cooling capacity (COP). In [108], the design variables were the leg length, the leg area and the number of legs, while in [109], the thermoelectric cooler considered was two-stage, and therefore, the design variables were the electrical current applied to colder TEC, the electrical current applied to hotter TEC, and the ratio of thermocouples between the two stages. Arithmetic crossovers were used by the GA.

3.2. Conduction heat transfer systems

In this section, we present a series of articles in which conduction-dominated heat transfer systems are optimized, and for which the modeling involves solving a diffusion equation. For example, the shape of fins can be optimized by GAs and topological optimization can be achieved by GAs to produce conduction pathways in a system. Compared to Section 3.1, the modeling of these systems is thus considered more “heavy” to some extent because every objective function evaluation requires solving a differential equation on the domain of interest.

3.2.1. Design of fins

Fins are used to increase the heat transfer surface area in various systems. The design of fins is an important problem in heat transfer and thus, has attracted a lot of attention, see Table 5. Many fin shapes are available (e.g., pin fins, annular fins and straight fins), and practical constraints should be considered in the design (e.g., cost, mass and manufacturability). In the last years, several authors have been using GAs to optimize fin shapes.

A 2D fin equation is solved by Fabbri in [110]. The objective is to maximize with a GA the effectiveness by varying the fin shape which is characterized by the thickness at $n + 1$ points (n took values up to 5). A constraint on the maximal and minimal thickness allowable was introduced.

The optimal fin shape that ensured a uniform longitudinal heat flux was computed in [111]. With a 1D fin model, the design variables were the radii of the circular cross-sections along the centerline. Up to 10 positions (i.e., 10 radii values) were optimized. In 2D, B-spline were used to parameter the fin shape. The ordinates of the poles (up to 12) were the design variables. The bounds of the design variables were continuously reduced as the generations went by, which was found to accelerate convergence. Ninety-nine individuals evolved for 25 generations.

A system of fins has been optimized in [112]. A 1D fin was used for each trapezoidal fin attached to a tube. The authors developed correlations which were used by the GA to estimate the performance, rather than calculating it explicitly with the model. The figure of merit was the heat transfer rate per unit of mass (to maximize) or the entropy generation rate (to minimize). Duct spacing, duct length, fin height, fin thickness and number of fins were the design variables. Each design was represented by a string of 32 bits.

The design of a stacking of micro-channel has been addressed in [113]. The GA minimized the overall thermal resistance for different pumping power. 1D or 2D fin equations were solved to evaluate the objective function. The parameters determined by the GA were the fin thickness, channel width and channel height.

Fabbri maximized the Nusselt number of internally finned tubes [114]. 2D laminar fully developed flow and temperature were solved with imposed heat flux at the outside wall. The design parameters were the wall thickness, the polynomial function

coefficients of the fin, and the center solid circle radius. The number of fins, solid–fluid thermal conductivity ratio, pressure drop and polynomial order (fin shape) were imposed. Several optimal shapes have been generated depending on these parameters. Similarly in [115], Fabbri maximized the heat transfer per unit of tube length or surface for a given weight and for a given hydraulic resistance. If after reproduction, the fin was too thin or too thick, the shape parameters had to be resized. The genetic algorithm was stopped after 50 generations because improvement was no longer observed. The approach was also extended to rectangular channels with symmetrical fins [116] and asymmetrical fins [117]. Fabbri noted a 95% increment of the optimized finned tube performance compared to simpler fins.

3.2.2. Other examples of designing problems based on conduction equation

Other examples of designing with GAs for conduction dominated problems are listed in Table 6. In order to cool a 2D body, a volume-to-point high conductive pathways in contact with a cold patch on the periphery has been designed in Ref. [50]. The objective was to minimize the hot spot temperature. Each chromosome was a vector of the highly conductive cells (not binary) and the maximal number of components (i.e., highly conductive cells) was 44. Other optimization approaches, simulated annealing (SA) and bionic optimization (BO), were used and compared to GAs. All approaches yielded similar results, although GA was slower to converge. Pedro et al. [118] designed a dendritic architecture with a GA for minimal hot spot temperature. The design variables were the bifurcation angle, the length of the first branch, the thickness of the first branch, and the reduction ratio between successive branches.

Goupee and Vel [119] optimized the fraction of composite material in 8 volume elements. 2D conduction equation was solved with the thermoelasticity problem to determine constraints. The RCGA minimized the residual stress under thermal and mass constraints or minimized mass under thermal and stress constraints. The GA used a tournament based selection. This allowed for the constraints not to be considered as penalty to the objective because the tournament selection always selects individuals that respect constraints over those who do not. Niching adaptation [38] has been considered to prevent the GA to fall into a local minimum by limiting the elimination of very unique and different individuals through the tournament selection process.

The shape of cooling channels to achieve a fixed temperature on the edge of a domain considering a gas temperature distribution at the boundary was designed with a GA in [120]. The GA minimized the error between given and computed temperature at the edge of the surface, and therefore, a 2D conduction equation with the point heat sink method was solved for each design tested. The two coordinates and the strength of each heat sink were the design variables. Different settings of the GA were studied, but the following was retained: crossover rate of 0.6, population of 50, and mutation rate of 0.2%. For example, a population of 10 individuals trapped the GA into local a minimum, inducing premature convergence.

Ref. [121] found the optimal design of a plastic window frame with air chambers and steel stiffeners with a BCGA, resulting in heat loss reduction of up to 30%. The objective function has been defined as minimum heat loss subject to a constraint of prescribed stiffness and weight of the steel insert. The heat loss is calculated with a 2D conduction equation. Contractions/expansions/translations of the air cavities, and deformations of the steel insert were to optimize, and were parameterized via the coordinates of control points. Different settings of the GA were investigated. The ones leading to the fastest convergence were: 30 individuals per population, mutation probability of 15%, probability of crossover of 50%.

The shape of a heat dissipater was designed by a GA in [122] either to maximize the heat transfer under total surface constraint or to minimize the heat transfer surface under constraints of temperature and stress. The authors solved a coupled conduction–radiation–elasticity problem using a boundary element procedure. The RCGA used 20 individuals of 14 genes each. The efficiency of three types of mutations (i – uniform: equal chance for a gene to take any value, ii – boundary: the gene receives a limit value; iii – Gaussian: probability distribution) and three types of crossovers (i – simple: gene interchanging; ii – arithmetical: interpolation between parents' gene; iii – heuristic: extrapolation following trends) was tested by realizing several parameter combinations. The best convergence was achieved with the Gaussian mutation and the simple crossover. The selection of the individuals inserted in the mating pool was based on a weighting value that took into account the fitness value and a selection pressure parameter. This parameter somehow controlled the best/average ratio of the weighting values attributed to each individual. “The selection pressure is the degree to which the better individuals are favored: the higher the selection pressure, the more the better individuals are favored” [122]. A cloning strategy also introduced elitism.

3.3. Design of thermofluid systems

Traditionally designing thermofluid systems with CFD is accomplished by performing an analysis over a very limited number of designs due to the high computational costs. With the constant improvement of computational resources, though, it is now possible to design thermofluid systems in a more efficient and rigorous way, for example with GAs (Table 7). As noted by Nobile et al., “there are no fundamental reasons, apart from computational costs and modeling accuracy issues, which prevent the application of the methodology [i.e., GAs] to more complex geometries, and more complex physics, such as, for example, three-dimensional channels, and unsteady or turbulent flow regimes” [123]. Nevertheless, so far most of the work relying on GAs for thermofluid optimization has been limited to simple geometries (e.g., channels) and models (e.g., 2D laminar and steady-state flow). Certainly we can expect to see more applications of GAs for the optimization of complex thermofluid systems in a near future.

Wildi-Tremblay and Gosselin [124] optimized a stacking of porous layers used as a heat sink. The flow was parallel to the heat-generating plate. Porosity and material composition in each layer were the design variables. The use of the GA was interesting for material selection. The hot spot was minimized and cost and mass constraints were considered via a penalty on the objective function when the constraints were violated. A 2D temperature equation was solved to determine the hot spot temperature of a design, and the velocity profile was known based on algebraic expression. The GA was also able to remove outer layers when they were useless. A similar problem, but in natural convection, was proposed by Villemure et al. [26]. In that case, the velocity profile was calculated with an iterative procedure. Some neighbouring designs (i.e., designs with small variations compared to an initial design) were generated and evaluated after crossovers (local search) to enhance repeatability and precision. Leblond and Gosselin [27] implemented a two temperature model (fluid and solid temperatures) to relax the local thermal equilibrium assumption of Refs. [26,124]. This new model allowed optimizing the pore diameter in each layer, in addition to the porosity and material. Local search was also considered. More recently, Tye-Gingras and Gosselin [125] developed a procedure to design a porous medium heat sink with the flow impinging on the hot surface (rather than parallel to the surface). Furthermore, a fin and a deflector were added. The GA optimized 14 design variables (porosity and material in 4 layers,

fin height, fin thickness, fin material, deflector height, deflector length and heat sink aspect ratio).

Back in 1994, Queipo et al. [8] ordered eight different discrete heat-generating elements in a channel to minimize failure rate (which depends on the temperature of the heaters). 2D CFD with a relatively coarse mesh was considered to estimate the temperature. The design was characterized solely by the way in which the 8 patches were placed on the channel wall (order). The GA converged 11 times faster than random search. Only 7 individuals per population evolved for few generations with elitism. da Silva and Gosselin maximized the thermal conductance of a channel with discrete heaters [126]. A 2D CFD model with laminar flow was used to evaluate the objective function. Up to 20 heater positions and heater strengths, as well as the channel breadth, were optimized (41 design variables) with a GA counting 100 individuals per population. The GA allowed increasing the number of design variables significantly compared to previous works that considered only few heaters. Ref. [127] also maximized the global conductance, but for a cavity, by positioning heat patches on its wall. Again, a 2D CFD model was used. Up to three design variables (e.g., patch positions) were considered. Due to high computational cost, a micro-GA algorithm with five individuals per population was used. Only 2% of the total number of possible designs were simulated by the GA.

The design of the shape of the tubes in a tube bank was the purpose of Ref. [128]. A 2D laminar flow was assumed, and two objectives were considered simultaneously, i.e., minimize the temperature and the pressure drop between the inlet and outlet. The Pareto front was determined. The optimization with the GA included four shape parameters. Thirty individuals per population and a mutation rate of 1% were considered.

In 1996, Schmit et al. [129] studied a compact high intensity cooler (CHIC). The design was optimized by a binary GA either for minimizing thermal resistance, pressure drop or a function combining both. Ten design variables were considered: number of slots per plate, number of holes per slot in x -direction, and in y -direction, number of orifice plates or fins between inlet and outlet, orifice plate thickness, space thickness, target spacer thickness, target cover thickness, hole diameter, and finally ratio of slot width to width conduction bus bar. The GA generated a better design than that reported in literature, based on “empirical designing”.

Nobile et al. [123] optimized a periodic channel with a multi-objective GA (MOGA-II). The flow was assumed to be fully developed, laminar and in a steady-state. The geometry of the channel was parametrized by means of non-uniform rational B-spline (NURBS) and their control points represented the design variables (up to 9 points). The Nusselt number was maximized and the friction factor was minimized. Directional crossovers were used. The population was made of 50 individuals.

A micro-heat exchanger has been designed in [130] with a multi-objective GA. The objectives were to minimize the pressure drop and maximize the heat transfer. Ten design variables (with 20 bits each) represented the separator shape. Each population was made of 100 individuals. Laminar, 2D, steady-state flow was assumed, and the objectives were evaluated with a CFD code.

The heat transfer was maximized by optimizing a wall corrugation profile under fixed pressure drop and volume of corrugated wall in Ref. [131]. Four points for the shape of the wall have been determined by the GA. Increment of up to 30% was achieved by the GA compared to a flat wall channel. During reproduction, random errors uniformly distributed between -10% and $+10\%$ of the parameter values were introduced to enhance the exploration of the design space. The number of individuals was set to 12.

Following [112], [132] considered a horizontal channel with vertical and rectangular fins mounted on the outside of the channel. They performed 130 2D CFD simulations to establish correlations that were later used by a standard GA to minimize the total

entropy generation rate or maximize the Nusselt number. Four design parameters were considered (channel length, channel spacing, fin height and number of fins).

In order to reduce the computational time, [133] recently considered an artificial neuron network (ANN) to approximate the objective function. A series of CFD simulations were performed to calculate the heat transfer efficiency of an air-to-water heat exchanger with the following eight design variables: length of air channel, height of water channel, width of air channel, height of air channel, width between air channels, thickness of wall that separates the water and air channels, width of water channel, and capacity ratio. Once the simulations have been performed and the ANN has been trained, the optimization with the GA proved to be very fast (~ 5 min) because the GA used the approximation of the objective function as provided by the ANN rather than having to perform a CFD simulation.

In [134], a BCGA was used to minimize the warpage of a thin shell plastic part. A series of numerical simulations (pressure, flow and temperature were solved) were performed for various x , y and z dimensions (i.e., the three design variables) to calculate the warpage. Then, a response surface methodology was used to approximate the design space. The GA used that approximation to optimize the design with 16 bits per design variable, and 50 individuals per population. The mutation rate was 10%. The GA reduced the warpage of the initial design by 40%.

The design of a chimney with a ribbed isothermally heated wall on one side and a smooth adiabatic wall on the other side has been performed in [135]. A design of experiments with 6 variables and 200 simulations was performed with a CFD commercial solver. Then a response surface modeling (RSM) was used to approximate the design space. The multi-objective GA maximized the channel averaged heat transfer coefficient and mass flow rate by adjusting six variables characterizing the ribbed channel geometry. The GA included 15 individuals per population that evolved for only 30 generations. However, after the GA was stopped, the simplex algorithm was applied to finesse the designs.

GAs were combined with CFD to optimize indoor environment (PMV human model method) in a two-step procedure [49]. First, coarse meshes were used and simplifying assumptions were invoked to perform optimization for identifying the most promising designs. The BCGA varied the office configuration, the location of the cooling panel and of the supply inlet, the radiative panel type, the surface temperature of the panel and the width of the supply inlet. Secondly, finer mesh and refined modeling was considered for evaluating better the 11 best individuals identified in the first step with the GA.

3.4. Radiation

The modeling and design of systems dominated by radiation can be a complex task. In particular, in the presence of participating media and with a coupling with convection or conduction, the models to evaluate the performance of a design become quite heavy. Therefore, only a very limited number of works with GAs have been found (Table 8). Designing the shape of a 2D radiative enclosure was the topic of [136]. The objective was to minimize the difference between desired and estimated heat flux profile over the designed surface. The radiative transfer equation with discrete transfer method was used to determine the error. An elitist BCGA varied the enclosure shape which was parameterized with rational B-spline curves. The two coordinates of the position of 6 control points (i.e., 12 design variables) were established by the GA. Then, the weights in the B-spline expression for fixed control points were optimized. This design problem could actually be seen as an inverse problem, which is the object of the next section.

4. Inverse heat transfer

The second class of heat transfer problems that have been using GAs is constituted by inverse heat transfer problems. Fifteen percent (20/132) of the reviewed articles fell into that category. Inverse heat transfer typically aims at determining material properties (e.g., thermal conductivity, heat capacity and specific extinction coefficient) or boundary conditions (e.g., heat transfer coefficient, heat flux and emissivity) based on measured values of temperature and/or heat flux in a system [137]. Such problems could be formulated as optimization problems. The objective is to minimize the error between measured values (e.g., temperature) and predicted values based on the estimated properties/BCs. Inverse problems are known to be ill-posed, i.e., that existence, uniqueness and stability of the solution are not satisfied under small changes of the input data (measurements).

Different optimization techniques have been used in the past to minimize the error between the measured and predicted data. GAs have also been used to perform the error minimization task in several heat transfer systems. Successes were achieved even though long computational times are often reported. In these problems, the equivalent to the design variables that were optimized by GAs in Section 3 are the properties/boundary conditions themselves. The GA will make an initial population of properties/BCs evolve in order to find those leading to the results as close as possible to the measurements. So far, only relatively simple systems have been solved with GAs, mainly in conduction and radiation heat transfer.

4.1. Inverse radiative heat transfer

A list of inverse radiation heat transfer problems addressed with GAs is presented in Table 9. In 1996, smoke parameters (refraction index, absorption coefficient and particle mean diameter) were determined by a standard BCGA in [138] but relatively few information on the GA was presented. More recently, [28] minimized the error between estimated and desired heat fluxes on up to 10 sub-surfaces of a cylindrical enclosure with gray and diffuse surfaces. The medium in the enclosure was assumed to be non-participating. The objective was to determine the temperature and emissivity on each sub-surface. Calculation times were long, but the resulting errors on the estimated emissivities and temperatures were relatively small, 1.3% and 4.3%, respectively. Local search was considered at the end of the GAs to improve the results.

Ref. [51] compared different algorithms for solving the inverse problem consisting in determining simultaneously two radiative properties (absorption coefficient and scattering coefficient) and two surface emissivities for a participating medium in-between two long parallel plates. The algorithm chosen was a binary micro-GA using 8 individuals per population. The GA was found to be slower than other procedures (Levenberg–Marquardt algorithm, artificial neural network, and the Bayesian algorithm), but more robust. In Ref. [139], Li and Yang determined the scattering albedo, the

optical thickness and the phase function with a GA, in order to minimize the discrepancies between measured and estimated intensities. Even though the first two parameters were fairly well identified by the GA, some difficulties were encountered for estimating the phase function.

4.2. Inverse conduction heat transfer

Table 10 lists inverse conduction heat transfer problems in which GAs were used. Jones et al. [140] used GAs to spot inhomogeneities. A conduction heat transfer inverse problem was solved to determine the thermal conductivity mapping based on the temperature measurement in a 2D surface. A standard BCGA minimized the error between measured and estimated temperatures. Successive zooming was applied around the identified inhomogeneities to finesse the conductivity map. Twenty-five blocks were considered, i.e., the thermal conductivity in 25 zones was searched. The GA ran with 200 individuals for 5 generations, first with a low resolution, and then with a higher resolution around the inhomogeneity. Even though the GA identified properly the conductivity mapping, it proved to be more computationally intensive than other inversion methods (e.g., linearization technique).

A procedure for solving inverse heat conduction problems was developed in [141]. The thermal conductivity and volumetric heat capacity were assumed to vary linearly with the temperature, i.e., $k = K_1 + K_2T$ and $\rho c_p = K_3 + K_4T$, and the four coefficients K_1 , K_2 , K_3 and K_4 were determined by the GA for a 1D problem, in order to minimize the error between measured and estimated temperatures. Performances of GA were compared with an ANN trained with 250 pairs of input–output obtained by the computed direct task. GA showed to be slower but more precise, achieving errors smaller than 0.1% on the temperature profile, against more than 1% with the ANN. However, both methods had significant errors on one or two of the four coefficients K .

Similarly, [142] proposed a method relying on GAs for characterizing a spatially dependent thermal conductivity, $k(x)$. A polynomial fit was considered for k , with up to 9 least-square points. The method exhibited a great robustness. In [143], Orain et al. determined the thermal conductivity and two contact resistances of a thin-film with a GA that minimized the difference between measured and calculated temperatures. Compared to the Gauss linearization method and the parameteric study, the GA allowed a more accurate simultaneous estimation of the parameters.

In [144,145], Garcia and Scott used an extended elitist GA to optimize a design of experiment to estimate the thermal properties of composite materials. Sensor positions, input heat flux duration and surface area were the design variables to maximize the precision of the experiment (D-optimal criterion). The range of each design variable was determined from an initial random search. The initial population in the GA was built by conserving the best designs of successive random search. After crossovers, the next generation was formed at 80% from the best among

Table 9
Summary of the GAs used for inverse radiative heat transfer.

Article	Problem			GA						
	Objective	Model	No. of variables	Single/multi	Bin/real	Pcross	Pmut	Nind	Maxgen	Elitism
[138]	Min. err. experimental vs. computed	Analytical	3	S	B	0.6	0.01	101	N/A	N/A
[28]	Min. err. desired vs. computed	Integral equation of the second kind	10	S	R	N/A	N/A	10	100	Y
[51]	Min err. experimental vs. computed	Radiative transfer equation	4	S	B	0.5	0.02	8	400	Micro-GA
[139]	Min. error on intensities	Radiative transfer equation	Up to 22	S	N/A	N/A	0.005	2000–5000	N/A	N/A

Table 10

Summary of the GAs used for inverse conduction heat transfer.

Article	Problem			GA						
	Objective	Model	No. of variables	Single/multi	Bin/Real	Pcross	Pmut	Nind	Maxgen	Elitism
[141]	Min. error on T	1D-transient heat conduction	4	S	N/A	N/A	N/A	32	500	Y
[142]	Min. error on T	2D conduction	N/A	S	B	N/A	N/A	N/A	200–750	N/A
[144]	Max. D-optimal criterion	1D, 2D conduction	N/A	S	R	N/A	N/A	50–200	10–200	Y
[145]	Max. D-optimal criterion	1D, 2D heat transfer	N/A	S	R	N/A	N/A	50–800	10–20	Y
[146]	Min. error on T	1D, 2D conduction	8	S	N/A	N/A	No	50	1% variation on the 3 last generation	Y
[147]	Min. error on T	1D conduction and Radiation	3	S	B	0.5	0.02	100	100	N/A
[148]	Min. error on K	1D conduction and radiation	5	S	N/A	N/A	N/A	N/A	N/A	N/A
[149]	Min. error on T	3D conduction	3	S	N/A	N/A	N/A	N/A	N/A	N/A
[150]	Min. error on T	1D conduction	50	S	N/A	N/A	N/A	32	500	Y
[151]	Min. error on heat flux and lag	1D conduction	3	S	N/A	N/A	N/A	N/A	Diff. of 0.0001 between the 2 last generation	N/A
[152]	Min. error on heat load	1D conduction – RC model	4	S	N/A	N/A	N/A	N/A	Diff. of 0.0001 between the 2 last generations	N/A
[153]	Min. error T	RC thermal model	N/A	S	B	N/A	N/A	N/A	N/A	N/A
[154]	Min. error on boundary voltage	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
[140]	Min. error between measured and calculated temperatures	2D conduction	25	S	B	0.6	0.01	200	5	N/A
[143]	Min. error between calculated and measured T	1D conduction	3	S	N/A	0.9	N/A	300	200	Y

children and parents, and at 20% from randomly generated designs for which the design variable range was based on the elites of the population. Test cases were considered. In case #1, the conductivity perpendicular to fibers, the in-plane conductivity, and the volumetric heat capacity of a composite material were determined (conduction). In case #2, radiative and thermal properties of an insulator (conductivity, heat capacity, extinction coefficient, scattering albedo and volumetric heat capacity of the heater) were determined. Box-Kanernasu method (gradient based approach) resulted in non-convergence in these cases, but the GA was able to estimate the properties. Similarly, Hanuska et al. [146] matched experimental and theoretical data by optimizing through-the-thickness volumetric heat capacity and through-the-thickness and in-plane conductivity of a composite material.

In addition to [145], combined inverse conduction and inverse radiative heat transfer with GAs was also studied by Verma and Balaji [147] and Daryabeigi [148]. In [147], three properties (surface emissivity, optical thickness and conduction radiation parameter) were to be determined based on temperature measurements with a 1D model. The results provided by the GA were not so good: the error was high when the input data were noisy. In [148] the properties to estimate were four (specific extinction coefficient, albedo of scattering, backscattering fraction and solid conduction exponent term), also with a 1D model. In that case, results were good as the error between measured and numerical data was generally minimized under the 7.5% experimental uncertainty. Unfortunately, in that case, the settings of the GA were not given to compare with [147].

Boundary conditions determination is an important class of inverse heat transfer problems. Liu et al. [149] sought to estimate the convection coefficient h on all the faces of an electronic package. 3D conduction with reduced-basis method was combined with FEM. Modifications to standard GAs were investigated. The number of individuals was reduced when progressing in the GA (micro-GA).

A local search between generations proved to reduce the number of generations required for convergence.

The determination of a transient heat transfer coefficient $h(t)$ in a 1D conduction problem was addressed in Ref. [150]. The objective was to minimize with the GA the error between estimated and measured temperature profiles. A penalty was added when $h(t)$ varied too abruptly (regularization). The easiness of implementation was noted. The authors noted that after 500 generations, solutions stopped to improve. The number of h -values determined was 50 (i.e., h at 50 times).

Conduction in building walls was represented by 3R2C, 2R2C and R2C thermal circuits, respectively, in Refs. [151–153]. GAs were used to determine the values of the capacitances and the resistances of the model, by minimizing the error between measurements and predicted values of heat fluxes and phase lags.

Adaptive mesh grouping method based on fuzzy-GA was introduced to reduce the image reconstruction time significantly in [154]. Resistivity in a domain was found based on measurements of boundary voltage.

4.3. Inverse convection heat transfer

Even though Ref. [155] is focused mainly on fluid mechanics, it provides an interesting view on how GA could be used for determining the best settings of a complex CFD model, which is certainly useful in convection heat transfer (Table 11). Three constants of a turbulence model have been determined by a BCGA in order to maximize the agreement between the CFD predictions and the experimental data. Reynolds-averaged Navier–Stokes equations were considered, with a k - ϵ model. A large number of local minima were present which reinforces the interest of GAs for this kind of problems. Each constant to determine was coded with 10 bits, for a total of 30 bits per design. Different population sizes were tested, and a population of five individuals was retained.

Table 11
Summary of the GAs used for inverse convection heat transfer.

Article	Problem			GA						
	Objective	Model	No. of variables	Single/multi	Bin/real	Pcross	Pmut	Nind	Maxgen	Elitism
[155]	Max. experimental-modeling agreement	3D CFD	3	S	B	0.5	0	5	150	Y

5. Fitting and estimation of model parameters

The development of correlations or relations based on a series of experiments or numerical simulations is often complicated. Authors have used GAs to minimize the error between a set of data and a correlation that one wants to develop, by finding the appropriate fitting coefficients (Table 12). Similarly, as mentioned in the previous section, the empirical parameters involved in models (e.g., building models and turbulence models) can be determined by a GA that compares simulations and experimental data. (10/132 = 8% of the papers).

Pacheco-Vega et al. [156] found a good fitting for the heat transfer coefficient in a heat exchanger with a GA. Four unknowns (fitting coefficients for the Nusselt in and over the tubes) coded with a total of 120 bits per individual were determined through regression analysis. They compared the GA with other optimization methods (i.e., simulated annealing (SA) and interval analysis (IM)). GA and SA were faster than the IM because they do not require information about the derivatives of the objective function. An advantage of GA over SA is that it produces a set of possible solutions at each iteration. However, noted the authors “GAs are not guaranteed to obtain the global optimum but only the region in which it is located with high probability. SA, on the other hand, is probabilistically, though not deterministically, guaranteed to find the optimum. The major virtue of IMs is that, unlike GAs and SA, they guarantee that the global optimum has been found.” However, IMs were computationally expensive to implement and to run.

The heat transfer and pressure drop of three types of shell-and-tube heat exchangers (one with conventional segmented baffles and two with continuous helical baffles) were measured experimentally with water (tube side) and oil (shell side), [157]. The GA determined the appropriate coefficients of the correlations by minimizing the error of the correlations with respect to the experimental data. Two coefficients for the heat transfer coefficient, and two other for the pressure drop were determined. Each coefficient was coded with 30 bits. Tournament selection, uniform crossover, and one-point mutation were selected. Niching and elitism were adopted. Twenty individuals evolved for 1000 genera-

tions. The settings of the GA were based on Refs. [158–159]. Excellent results were achieved by the GA.

A correlation has been developed for the heat transfer coefficient on the exterior wall of a building with respect to wind velocity by minimizing with a BCGA the error of the correlation with respect to experimental data [160]. Parameter estimation in building thermal model is often a difficult task. By finding a best fit of the frequency response, three or four model parameters (thermal resistance and thermal capacitances) were determined by a GA in [161]. The algorithm was validated with a real case. In [21], a multi-objective GA (MOGA) selected the most relevant model inputs and the topology of a radial basis function artificial neural network (RBF-ANN) that predicted building temperature based on experimental data. Sixteen objectives were defined and classified in three categories: model complexity, model performance and model validity. In other words, the GA had to find the most 2–30 relevant model inputs out of 60 possible ones and choose the number of neurons in the RBF-ANN in order to obtain the best fitting with experimental data with the lowest structure complexity. Ten percent of random immigrants, a survival rate of 50% and a selection pressure of 2 were among the parameters chosen for the GA. Random immigrants were new individuals reinserted in each population and the survival rate was the proportion of the offspring population that survived and reproduced at each generation. The selection pressure coefficient amplified or reduced the weighted fitness gaps between individuals in order to operate a more or less drastic selection process. Ozturk et al. [162] minimized the error between actual energy inputs of the residential and commercial sectors in Turkey and the prediction of a model. Twenty-seven weighted factors associated to the population, GDP, exports/imports, residential production, cement production and sales of house appliances were varied by a BCGA with a crossover probability of 80% and a mutation rate of 2%. The population was made of 100 individuals. In [163], the discrepancy between actual and estimated energy consumption was minimized by a BCGA. Ten weighting factors were determined by the GA with a population of 20, evolving during 250 generations. The crossover and mutation probability were 50% and 5%.

Table 12
Summary of the GAs used for fitting and model parameter estimation.

Article	Problem			GA						
	Objective	Model	No. of variables	Single/multi	Bin/real	Pcross	Pmut	Nind	Maxgen	Elitism
[156]	Min. error between correlation and experiment	Analytical	4	S	B	1	0.03	30	End at 2959	N/A
[157]	Min. error between correlation and experiment	Analytical	4	S	B	0.5	0.005	20	1000	Y
[160]	Min. error between correlation and experiment	Analytical	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
[161]	Min. integrated err. exp. vs. predicted frequency response	Analytical	3–4	S	N/A	N/A	N/A	N/A	N/A	N/A
[21]	Min. complexity min. different err metrics	Analytical (ANN)	31	M	0.7	N/A	100	100	N/A	N/A
[162]	Min. err. actual vs. predicted	Analytical	27	S	B	0.8	0.02	100	450	N/A
[163]	Min. err. actual vs. predicted	Analytical	10	S	B	0.5	0.05	20	250	Y
[164]	Min. err. actual vs. predicted	Analytical	N/A	S	N/A	N/A	N/A	N/A	N/A	N/A
[165]	Min. err. actual vs. predicted	Analytical	7	S	R	0.85	0.1	20	2000	N/A
[166]	Min. err. actual vs. predicted	Analytical	4	S	B	0.5	0.05	32	300	N/A

GA was used to select most effective molecular descriptors that affect the values of the standard chemical exergy [164]. This work helped to find correlations for exergy by minimizing the error of the correlation with experimental data.

Seven parameters of empirical equations in PEM modeling have been determined by a RCGA in [165]. The authors wanted these empirical parameters (scaled to vary between 0 and 1) to minimize the difference between experimental and theoretical V–I curves. The GA counted 20 individuals per population, and the mutation rate was 10%. Niching and simplex local search were also included in the optimization process. As described in Section 2, niching helps maintaining the population diversity, and local search helps the GA to finesse the search in a region of the variable space.

Balaji et al. [166] found wall function from correlations in the turbulent natural convection parameter estimation problem. Four unknown coefficients were determined with a standard BCGA to minimize the difference between measured and predicted temperatures. A comparison of the GA with the Levenberg–Marquardt algorithm showed nearly identical results for both methods. However, there was no comparison on the computational time required.

6. Other applications

In addition to the previous applications described above, a certain number of applications of GAs to heat transfer related problems are worth mentioning (Table 13). For example, GAs have been envisioned to solve matrix systems. Solving a matrix system of the form $Ax = b$ is a common task to perform in numerical heat transfer. It could be translated into an equivalent residual minimization problem. The GAs could thus be used to perform this minimization and determine the “optimal” x . 2D conduction problems were solved that way with a RCGA [167–168]. In [168], up to 196 unknowns (i.e., temperature at 196 nodes) were estimated. A “genetic engineering” operator (local search) is proposed in which the best individual in each generation is improved gene by gene. Elitism and wide range of population (10–1000 individuals) were tested. Elitism and local search were also included in the GA of [167]. Larger problems (up to 40,000 unknowns) were addressed with a parallelized GA taking advantage of 152 processors. Local search is performed as follows: immediately after the fitness evaluation, a search is sequentially performed at each gene locus across individuals for the allele with the smallest residual for that locus. Small populations were used (10 individuals). In [169], the authors used a BCGA to divide a domain into subdomains for parallel computing, by minimizing the communication between intersection nodes.

Phonon–phonon normal and Umklapp scattering processes were modeled with a genetic algorithm to satisfy energy and momentum conservation in [170]. Fitness was an indication of how well the ensemble satisfied momentum and energy conservations and was reflected by two parameters which are the residuals of the wave vector and the frequency. Twelve bits per phonon were used.

7. Conclusions

In this review, we presented the most recent publications discussing heat transfer related problems that were solved with genetic algorithms. As demonstrated above, GAs have been used increasingly by the heat transfer community, and could now be qualified as a “mature” optimization approach in our field. Therefore, one of the recommendations that we might formulate is that it might not be necessary at this point to describe extensively the GA procedure in new articles to be published, as has been done regularly so far. However, the present review has also revealed that many articles are not explicit as to what settings were used for their GAs. For the sake of repeatability, it would be useful to provide insights as to what settings were used, in particular:

- Is the GA binary/real, single/multi-objective, elitist/non-elitist?
- What is the design representation?
- What are mutation and crossover probabilities?
- What is the convergence criterion?

Another aspect that is largely under-verified is the repeatability of the GA as two runs with the same settings can potentially lead to different results.

We have seen that the complexity level of the problems considered so far covers a wide range. The complexity of a problem includes the complexity of the modeling (e.g., model based on analytical expressions vs. 3D-CFD model), and the number and type of design variables. The models considered in the body of work reviewed included mainly analytical expressions, but also conduction and diffusion equations, CFD or radiative transfer equations. The number of design variables is typically relatively low (i.e., ~ 5 and less). Even though some successes were also achieved with large numbers of design variables (e.g., 73 in [70] and 200 design variables in [73]), there is still a lot of work to accomplish to solve and improve the convergence of complex problems either because of the large number of variables or because of the heavy modeling while preserving the simplicity of GAs.

One promising avenue to do so is with parallel calculation which can result in important computational time reduction. Optimizations based on GAs are massively parallelizable because the fitness of many individuals can be evaluated simultaneously by different processors, e.g., see [128,167].

Another option to be examined more thoroughly is the use of approximations for the fitness function. In other words, in place of calculating in details the fitness of an individual (for example with a CFD simulation), one could use an approximation that yields faster results. Artificial neural networks (ANN) and response surface modeling (RSM) (see, e.g., Refs. [17–21,135]) are possible ways to create these approximations.

Finally, more work should be done to evaluate quantitatively and document the impact of the different GA settings and variations on the true performance of the algorithm for the kind of

Table 13
Summary of the GAs used for other applications.

Article	Problem			GA						
	Objective	Model	No. of variables	Single/multi	Bin/real	Pcross	Pmut	Nind	Maxgen	Elitism
[167]	Min. residual	2D conduction	Up to 40 000	S	R	Proportional selection	N/A	10	500	Y
[168]	Min. residual	2D conduction	196	S	R	SRS	N/A	10–1000	200	Y
[169]	Min. communications between intersection nodes	Algebraic	N/A	S	B	N/A	0.001	N/A	10000	N/A
[170]	Min. residual	Algebraic	N/A	S	B	N/A	N/A	N/A	N/A	N

problems that are of interest to the heat transfer community. In any case, modifications and improvements of GAs must try to preserve its facility of use.

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